Class IX Session 2023-24
Subject - Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. The abscissa of any point on y-axis is **[1]**

a) 1 b) any number

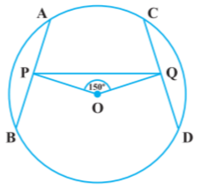
c) -1 d) 0

2. Area of an isosceles triangle ABC with AB = a = AC and BC = b is **[1]**

a) $\frac{1}{4}b\sqrt{4a^2-b^2}$ b) $\frac{1}{4}b\sqrt{a^2-b^2}$

c) $\frac{1}{2}b\sqrt{4a^2-b^2}$ d) $\frac{1}{2}b\sqrt{a^2-b^2}$

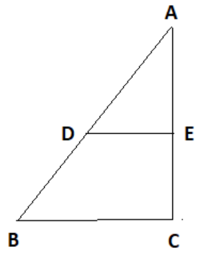
3. In Fig., AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD, respectively. If ∠POQ = 150º, then ∠APQ is equal to **[1]**



a) 60º b) 75º

c) 15º d) 30º

4. D and E are the mid-points of the sides AB and AC. Of △ABC. If BC = 5.6cm, find DE. **[1]**

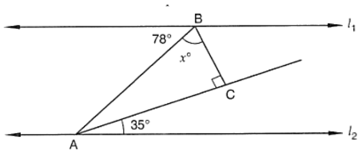


a) 2.8 cm
b) 3 cm
c) 2.9 cm
d) 2.5 cm

5. $(625)^{0.16} \times (625)^{0.09} =$ **[1]**

a) 625
b) 5
c) 125
d) 25

6. In figure, for which value of x is $l_1 || l_2$? **[1]**



a) 43
b) 37
c) 45
d) 47

7. The taxi fare in a city is as follows: For the first kilometer, the fare is ₹8 and for the subsequent distance it is ₹5 per kilometer. Taking the distance covered as x km and total fare as ₹y, write a linear equation for this information. **[1]**

a) y = 5x + 3
b) y = 5x – 3
c) x = 5y – 3
d) x = 5y + 3

8. If $x^2 + kx - 3 = (x-3)(x+1)$, then the value of ‘k’ is **[1]**

a) -3
b) 2
c) -2
d) 3

9. If $10^x = 64$, what is the value of $10^{\frac{x}{2}+1}$? **[1]**

a) 18
b) 80
c) 81
d) 42

10. In Triangle ABC which is right angled at B. Given that AB = 9cm, AC = 15cm and D, E are the mid-points of the sides AB and AC respectively. Find the length of BC? **[1]**

a) 13cm
b) 13.5cm
c) 12cm
d) 15cm

11. When simplified $(x^{-1} + y^{-1})^{-1}$ is equal to **[1]**

a) xy
b) x + y
c) $\frac{xy}{x+y}$
d) $\frac{x+y}{xy}$

12. The system of linear equations ax + by = 0, cx + dy = 0 has a non-trival solution if **[1]**

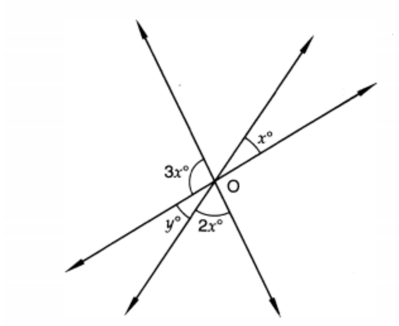
a) ad – bc = 0

b) ad – bc < 0

c) ad –bc = 0

d) ac + bd = 0

13. In Fig. the value of y, is **[1]**



a) 60°

b) 45°

c) 20°

d) 30°

14. $\sqrt[4]{\sqrt[3]{2^2}}$ is equal to **[1]**

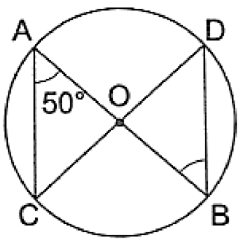
a) 2^{-6}

b) $2^{-\frac{1}{6}}$

c) $2^{\frac{1}{6}}$

d) 2^6

15. In the given figure, O is the centre of a circle. If ∠OAC = 50°, then ∠ODB = ? **[1]**



a) 50°

b) 60°

c) 75°

d) 40°

16. The point which lies on x-axis at a distance of 4 units in the negative direction of x-axis is **[1]**

a) (4, 0)

b) (-4, 0)

c) (0, -4)

d) (0, 4)

17. The positive solutions of the equation ax + by + c = 0 always lie in the **[1]**

a) 3rd quadrant

b) 4th quadrant

c) 2nd quadrant

d) 1st quadrant

18. If $f(x) = x^2 - 5x + 1$, then the value of f(2) + f(-1) is **[1]**

a) 2

b) 1

c) -2

d) -1

19. **Assertion (A):** If the diagonals of a parallelogram ABCD are equal, then ∠ABC = 90° **[1]**

Reason (R): If the diagonals of a parallelogram are equal, it becomes a rectangle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** Rational number lying between two rational numbers a and b is $\frac{a+b}{2}$. **[1]**

Reason (R): There is one rational number lying between any two rational numbers.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

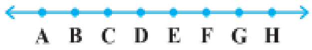
c) A is true but R is false.

d) A is false but R is true.

Section B

21. If P, Q, and R are three points on a line and Q is between P and R, then prove that PR - QR = PQ. **[2]**

22. Look at the Fig. Show that length AH > sum of lengths of AB + BC + CD. **[2]**



23. In which quadrant will the point lie, if : **[2]**

(i) The y-coordinate is 3 and the x-coordinate is –4?

(ii) The x-coordinate is –5 and the y-coordinate is –3?

(iii) The y-coordinate is 4 and the x-coordinate is 5?

(iv) The y-coordinate is 4 and the x-coordinate is –4?

24. Simplify the following by rationalizing the denominator : $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ **[2]**

OR

Simplify: $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}}-64^{\frac{2}{3}}\right]$

25. The largest sphere is carved out of a solid cube of side 21 cm. Find the volume of the sphere. **[2]**

OR

If the radius of the base of a right circular cone is halved keeping the height same, what is the ratio of the volume of the reduced cone to that of the original one?

Section C

26. Express $0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ **[3]**

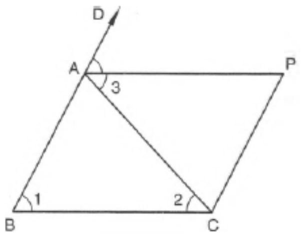
27. The production of oil (in lakh tonnes) in some of the refineries in India during 1982 was given below: **[3]**

Refinery:	Barauni	Koyali	Mathura	Mumbai	Florida
Production of oil (in lakh tonnes)	30	70	40	45	25

Construct a bar graph to represent the above data so that the bars are drawn horizontally.

28. In the figure, ABC is an isosceles triangle in which AB = AC. CP || AB and AP is the bisector of exterior ∠CAD of △ABC. **[3]**

Prove that (i) ∠PAC = ∠BCA and (ii) ABCP is a parallelogram.



29. Find solutions of the form x = a, y = 0 and x = 0, y = b for the following pairs of equations. Do they have any common such solution? **[3]**

5x + 3y = 15 and 5x + 2y = 10

30. Given below are the seats won by different political parties in the polling outcome of a state assembly elections: **[3]**

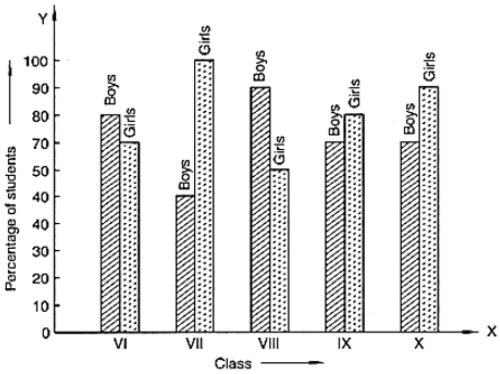
Political party	A	B	C	D	E	F
Seats won	65	52	34	28	10	31

Draw a bar graph to represent the polling results.

OR

The following bar graph shows the results of an annual examination in a secondary school.

Read the bar graph (Figure) and choose the correct alternative in each of the following:



i. The pair of classes in which the results of boys and girls are inversely proportional are:

a. VI, VIII

b. VI, IX

c. VIII,IX

d. VIII, X

ii. The class having the lowest failure rate of girls is

a. VII

b. X

c. IX

d. VIII

iii. The class having the lowest pass rate of students is

a. VI

b. VII

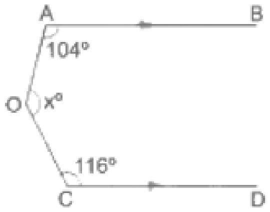
c. VIII

d. IX

31. Using factor theorem, factorize the polynomial: $x^4 - 7x^3 + 9x^2 + 7x - 10$ **[3]**

Section D

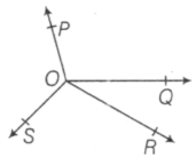
32. In the given figure, AB || CD and $\angle AOC = x^\circ$. If $\angle OAB = 104^\circ$ and $\angle OCD = 116^\circ$, find the value of x. **[5]**



OR

In the given figure, OP, OQ, OR and OS are four rays. Prove that

$\angle POQ + \angle ROQ + \angle SOR + \angle POS = 360°$.



33. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. **[5]**

34. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle. **[5]**

OR

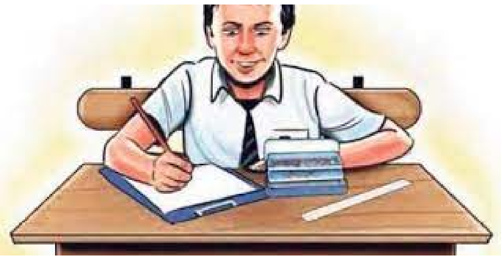
If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle.

35. Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $(x^2 - 1)$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ajay is writing a test which consists of ‘True’ or ‘False’ questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. Ajay knew answers to some of the questions. Rest of the questions he attempted by guessing.



(i) If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he answer correctly?

(ii) If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he guess?

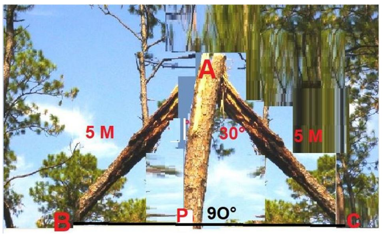
(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

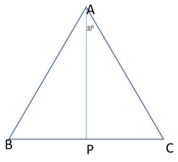
OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

37. **Read the text carefully and answer the questions:** **[4]**

In a forest, a big tree got broken due to heavy rain and wind. Due to this rain the big branches AB and AC with lengths 5m fell down on the ground. Branch AC makes an angle of 30° with the main tree AP. The distance of Point B from P is 4 m. You can observe that ΔABP is congruent to ΔACP.





(i) Show that △ACP and △ABP are congruent.

(ii) Find the value of ∠ACP?

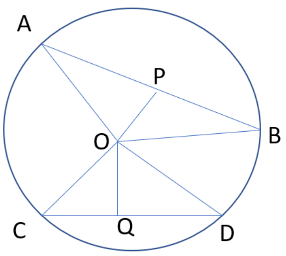
(iii) Find the value of ∠BAP?

OR

What is the total height of the tree?

38. **Read the text carefully and answer the questions:** **[4]**

Rohan draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.



(i) Show that the perpendicular drawn from the Centre of a circle to a chord bisects the chord.

(ii) What is the length of CD?

(iii) What is the length of AB?

OR

How many circles can be drawn from given three noncollinear points?

Solution

Section A

1.

(d) 0

Explanation: The abscissa of any point on y-axis is always zero. This means that this point hasn't covered any distance on x-axis.

2. **(a)** $\frac{1}{4}b\sqrt{4a^2 - b^2}$

Explanation: Here s = $\frac{a+a+b}{2} = \frac{2a+b}{2}$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

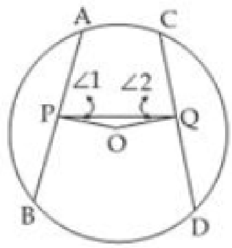
= $\sqrt{\frac{2a+b}{2}\left(\frac{2a+b}{2} - a\right)\left(\frac{2a+b}{2} - a\right)\left(\frac{2a+b}{2} - b\right)}$

= $\sqrt{\frac{2a+b}{2}\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2a-b}{2}\right)}$

= $\frac{b}{4}\sqrt{4a^2 - b^2}$

3.

(b) 75°

Explanation: 

As AB = CD

So, OP = OQ (equal chords are equidistant from the centre)

$\angle 1 = \angle 2$ (angles opposite to equal sides are equal)

$\angle 1 + \angle 2 + \angle \text{POQ} = 180^\circ$

$\angle 1 + \angle 1 + 150^\circ = 180^\circ$

$\therefore \angle 1 = 15^\circ$

Since APB is a line segement

$\therefore \angle \text{BPO} + \angle 1 + \angle \text{APQ} = 180^\circ$

$90^\circ + 15^\circ + \angle \text{APQ} = 180^\circ$

$\therefore \angle \text{APQ} = 75^\circ$

4. **(a)** 2.8 cm

Explanation: By using Mid-Point theorem,

DE = Half of BC

Hence, DE = 0.5 × 5.6 = 2.8 cm

5.

(b) 5

Explanation: $(625)^{0.16} \times (625)^{0.09}$

= $(625)^{0.16 + 0.09}$

= $(625)^{0.25}$ or $(625)^{\frac{1}{4}}$

But 625 = 5^4

So, $(5^4)^{\frac{1}{4}}$ = 5

6.

(d) 47

Explanation: Let if $l_1 || l_2$ and AB is tranverse to it

Then,

$\angle$PBA should be equal to $\angle$BAS (Alternate angles)

So if $l_1 || l_2$, then $\angle BAS = 70^o$

$\Rightarrow \angle BAC = 78^o - 35^o = 43^o$..(i)

Now, in $\triangle ABC$

$x^o + \angle C + \angle BAC = 180^o$

$\Rightarrow x^o + 90^o + 43^o = 180^o$

$\Rightarrow x^o = 180^o - 90^o - 43^o = 47^o$

$\Rightarrow x^o = 47^o$

So if $x^o = 47^o$ then $l_1 || l_2$

7. **(a)** y = 5x + 3

Explanation: Taxi fare for first kilometer = ₹8

Taxi fare for subsequent distance = ₹5

Total distance covered = x

Total fare = y

Since the fare for first kilometer = ₹8

According to problem, Fare for (x - 1) kilometer = 5(x - 1)

So, the total fare y = 5(x - 1) + 8

$\Rightarrow$ y = 5(x - 1) + 8

$\Rightarrow$ y = 5x – 5 + 8

$\Rightarrow$ y = 5x + 3

Hence, y = 5x + 3 is the required linear equation.

8.

(c) -2

Explanation: $x^2 + kx - 3 = (x-3)(x+1)$

=> $x^2 + kx - 3 = x^2 + (-3+1)x + (-3) \times 1$

=> $x^2 + kx - 3 = x^2 - 2x - 3$

On comparing the term, we get $k = -2$

9.

(b) 80

Explanation: $10^x = 64$

$\Rightarrow \sqrt{10^x} = \sqrt{64}$

$\Rightarrow (10^x)^{\frac{1}{2}} = 8$

$\Rightarrow 10^{\frac{x}{2}} = 8$

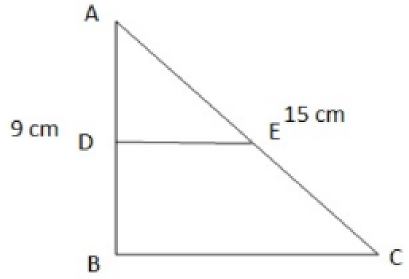
Now, $10^{\frac{x}{2}+1} = 10^{\frac{x}{2}} \times 10^1$

$= 8 \times 10 = 80$

10.

(c) 12cm

Explanation:



Applying pythagoras theorem in $\triangle ABC$

$AC^2 = AB^2 + BC^2$
$15^2 = 9^2 + BC^2$
$225 = 81 + BC^2$
$225 - 81 = BC^2$
$BC^2 = 144$
BC = 12 cm

11.
(c) $\frac{xy}{x+y}$

Explanation: $(x^{-1} + y^{-1})^{-1}$

$= (\frac{1}{x} + \frac{1}{y})^{-1}$
$= (\frac{y+x}{xy})^{-1}$
$= \frac{xy}{x+y}$

12. **(a)** ad – bc = 0

Explanation: The given system of equations has a non – trivial solution if: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0 \Rightarrow ad - bc = 0$.

13.
(d) 30°

Explanation: 3x + y + 2x = 180^o (Linear pair)

5x + y = 180^o (i)
From figure,
y = x (Vertically opposite angles)
Using it in (i), we get

5x + x = 180^o

6x = 180^o

x = 30^o
Thus,

y = x = 30^o

14.
(c) $2^{\frac{1}{6}}$

Explanation: $\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2)^{\frac{2}{3}}}$

$= (2)^{\frac{2}{3} \times \frac{1}{4}}$

$= (2)^{\frac{1}{6}}$

15. **(a)** 50^o

Explanation: ∠ODB = ∠OAC = 50^o (Angles in the same segment of a circle)

⇒ ∠ODB = 50^o

16.
(b) (-4, 0)
Explanation: Since it lies on x-axis so,ordinate will be zero,
Thus point will be (—4,0)

17.
(d) 1st quadrant
Explanation: The positive solutions of the equation ax + by + c = 0 always lie in the 1st quadrant
Because in 1st quadrant both x and y have positive value.

18. **(a)** 2

Explanation: f(x) = x^2 - 5x + 1

f(2) + f(-1)

= $(2)^2 - 5 \times 2 + 1 + (-1)^2 - 5 \times (-1) + 1$

= 4 - 10 + 1 + 1 + 5 + 1

= 12 - 10

= 2

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation: There are infinitely many rational numbers between any two given rational numbers.

Section B

21. From the given condition, we get the following figure

In the above figure, PQ coincides with PR - QR.

So, according to Euclid's axiom, "things" which coincide with one another are equal to 'one another'. We have,

PQ + QR = PR i.e. PR - QR = PQ.

22. From the given figure, we have

AB + BC + CD = AD [AB, SC and CD are the parts of AD] Here, AD is also the parts of AH.

By Euclid's axiom, the whole is greater than the part. i.e., AH > AD.

Therefore, length AH > sum of lengths of AB + BC + CD.

23. (i) II

(ii) III

(iii) I

(iv) II

24. $\frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$

(Multiplying the numerator and denominator by $5\sqrt{3}+3\sqrt{5}$)

$= \frac{30(5\sqrt{3}-3\sqrt{5})}{(5\sqrt{3})^2-(3\sqrt{5})^2} = \frac{30(5\sqrt{3}-3\sqrt{5})}{75-45}$

$= \frac{30(5\sqrt{3}-3\sqrt{5})}{30} = \frac{5\sqrt{3}-3\sqrt{5}}{1}$

OR

$64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$

$= (4^3)^{-\frac{1}{3}}\left[(4^3)^{\frac{1}{3}} - (4^3)^{\frac{2}{3}}\right]$

= $4^{\frac{-3}{3}}\left(4^{3\times\frac{1}{3}} - 4^{3\times\frac{2}{3}}\right) = 4^{-1}(4-4^2)$ [$\because (a^m)^n = a^{mn}$]

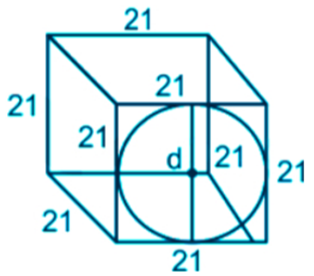
$= \frac{1}{4}(4-16) = -\frac{12}{4} = -3$ [using $a^{-1} = \frac{1}{a}$]

25. Given: Side of cube = 21 cm

Formulas used:

Volume of sphere = $\frac{4}{3}\pi r^3$

Calculation:



The largest sphere that can be carved out of a cube of side 21 cm will have the diameter equal to 21 cm.

Radius of sphere = $\frac{21}{2}$ cm

Volume of sphere = $\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$\Rightarrow 11 \times 21 \times 21$

$\Rightarrow$ 4851 cm^3

$\therefore$ The required result is 4851 cm^3

OR

Let the radius of the base and the height of the original cone be r and h respectively.

$\therefore$ Volume of the original cone (v_1) = $\frac{1}{3}\pi r^2 h$ . . . (1)

For the reduced cone

Radius = $\frac{r}{2}$

Height = h

$\therefore$ Volume of the reduced cone (v2) =$\frac{1}{3}\pi(\frac{r}{2})^2 h$

$= \frac{1}{4}(\frac{1}{3}\pi r^2 h) = \frac{1}{4}v_1$ . . . [From (1)]

$\therefore \frac{v_2}{v_1} = \frac{1}{4} = 1:4$

$\therefore$ The ratio of the volume of the reduced cone to that of the original one is 1 : 4

Section C

26. Let x = $0.4\overline{7}$ = 0.47777 ...

Multiplying both sides by 10 (since one digit is repeating), we get

10x = 4.7777 ...

⇒ 10x = 4.3 + 0.47777 ...

⇒ 10x = 4.3 + x

⇒ 10x - x = 4.3

⇒ 9x = 4.3

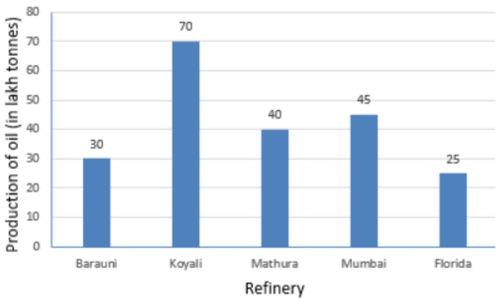
$\Rightarrow x = \frac{4.3}{9} = \frac{43}{90}$

Thus, $4.\overline{7}$ = $\frac{43}{90}$

Here p = 43

q = 90($\neq$0)

27. The production of oil (in lakh tonnes) in some of the refineries in India during 1982



28. **GIVEN** An isosceles △ ABC having AB = AC. AP is the bisector of ext ∠CAD and CP || AB

TO PROVE ∠PAC = ∠BCA and ABCP is a parallelogram.

PROOF

i. In △ ABC, we have

AB = AC [Given]

⇒ ∠1 = ∠2 [∵ Angles opposite to equal sides in a △ are equal] ....(i)

In a triangle, an exterior angle is equal to the sum of two opposite interior angles.

∴ In △ABC, we have

∠CAD = ∠1 + ∠2

⇒ ∠CAD = 2 ∠2 [using (i)]

⇒ 2 ∠3 = 2 ∠2 [∵ AP is the bisector of ext. ∠CAD ∴ ∠CAD = 2∠3]

⇒ ∠3 = ∠2

⇒ ∠PAC = ∠BCA

ii. We observe that AC intersects lines AP and BC at A and C respectively such that ∠3 =∠2 i.e., alternate interior angles are equal.

∴ AP || BC

But, CP || AB [Given]

Thus, ABCP is a quadrilateral such that AP || BC and CP || AB.
Hence, ABCP is a parallelogram.

29. $5x + 3y = 15$

Put x = 0, we get

$5(0) + 3y = 15$

$\Rightarrow 3y = 15$

$\Rightarrow y = \frac{15}{3} = 5$

$\therefore$ (0, 5) is a solution.

$5x + 3y = 15$

Put y = 0, we get

$5x + 3(0) = 15$

$\Rightarrow 5x = 15$

$\Rightarrow x = \frac{15}{5} = 3$

$\therefore$ (3, 0) is a solution.

$5x + 2y = 10$

Put x = 0, we get

$5(0) + 2y = 10$

$\Rightarrow 2y = 10$

$\Rightarrow y = \frac{10}{2} = 5$

$\therefore$ (0, 5) is a solution.

$5x + 2y = 10$

Put y = 0, we get

$5x + 2(0) = 10$

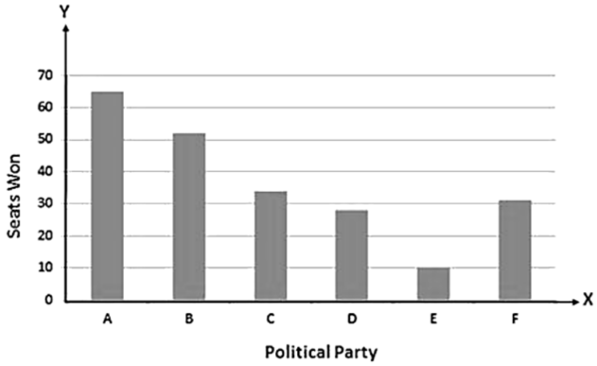
$\Rightarrow 5x = 10$

$\Rightarrow x = \frac{10}{5} = 2$

$\therefore$ (2, 0) is a solution.

The given equations have a common solution (0, 5).

30. The bar graph is given below:



OR

i. (b) VI, IX
ii. (a) VII
iii. (b) VII

31. The given polynomial is,

$f(x) = x^4 – 7x^3 + 9x^2 + 7x – 10$

The constant term in f(x) is 10

The factors of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

Let, $x – 1 = 0$

$\Rightarrow x = 1$

Substitute the value of x in f(x),then,we have,

$f(1) = 1^4 – 7(1)^3 + 9(1)^2 + 7(1) – 10$

$= 1 – 7 + 9 + 7 – 10$

$= 10 – 10$

$= 0$

(x – 1) is the factor of f(x)

Similarly, the other factors are (x + 1), (x – 2), (x – 5)
Since, f(x) is a polynomial of degree 4. Therefore, it cannot have more than four linear factor.
So, f(x) = k(x – 1)(x + 1)(x – 2)(x – 5)

$\Rightarrow x^4 - 7x^3 + 9x^2 + 7x - 10 = k(x-1)(x+1)(x-2)(x-5)$

Put x = 0 on both sides

0 – 0 + 0 – 10 = k(-1)(1)(-2)(-5)

– 10 = k(-10)

⇒ k = 1

Substitute k = 1 in f(x) = k(x – 1)(x + 1)(x – 2)(x – 5)

f(x) = (1)(x – 1)(x + 1)(x – 2)(x – 5)

= (x – 1)(x + 1)(x – 2)(x – 5)

So, $x^4 - 7x^3 + 9x^2 + 7x - 10$

= (x – 1)(x + 1)(x – 2)(x – 5)

Section D

32. Through O draw OE || AB || CD

Then, $\angle AOE + \angle COE = x^\circ$

Now, AB || OE and AO is the transversal

$\therefore \angle OAB + \angle AOE = 180^\circ$

$\Rightarrow 104^\circ + \angle AOE = 180^\circ$

$\Rightarrow \angle AOE = (180 - 104)^\circ = 76^\circ$ .....(1)

Again, CD || OE and OC is the transversal

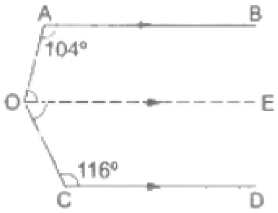
$\therefore \angle COE + \angle OCD = 180^\circ$

$\Rightarrow \angle COE + 116^\circ = 180^\circ$

$\Rightarrow \angle COE = (180^\circ - 116^\circ) = 64^\circ$ ....(2)

$\therefore \angle AOC = \angle AOE + \angle COE = (76^o + 64^o) = 140^o$ [from (1) and (2)]

Hence, $x^\circ = 140^o$

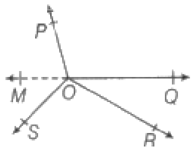


OR

Let us produce a ray OQ backwards to a point M, then MOQ is a straight line.

Now, OP is a ray on the line MOQ. Then, by linear pair axiom, we have

∠MOP + ∠POQ = 180^o ......(i)



Similarly, OS is a ray on the line MOQ. Then, by linear pair axiom, we have

∠MOS + ∠SOQ = 180^o ....(ii)

Also, ∠SOR and ∠ROQ are adjacent angles.

∴ ∠SOQ = ∠SOR + ∠ROQ ...(iii)

On putting the value of ∠SOQ from Eq.(iii) in Eq.(ii), we get

∠MOS + ∠SOR + ∠ROQ = 180^o ....(iv)

Now, on adding Eqs.(i) and (iv), we get

∠MOP + ∠POQ + ∠MOS + ∠SOR + ∠ROQ = 180^o + 180^o

⇒ ∠MOP + ∠MOS + ∠POQ + ∠SOR + ∠ROQ = 360^o ....(iv)

But ∠MOP + ∠MOS = ∠POS

Then, from Eq.(v), we get

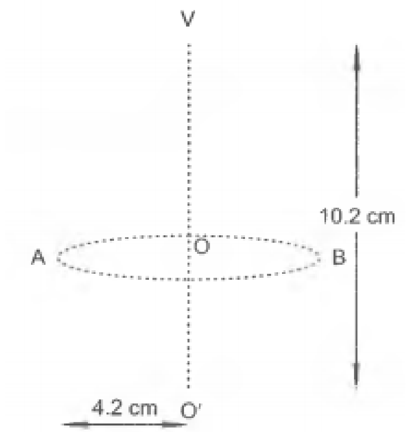
$\angle POS + \angle POQ + \angle SOR + \angle ROQ = 360^o$

Hence proved.

33. We have, VO' = 10.2cm, OA = OO' = 4.2cm

Let r be the radius of the hemisphere and h be the height of the conical part of the toy.

Then, r = OA = 4.2 cm, h = VO = VO' - OO' = (10.2 - 4.2) cm = 6 cm



Also, radius of the base of the cone = OA = r = 4.2 cm

$\therefore$ Volume of the wooden toy = Volume of the conical part + Volume of the hemispherical part

$= \left(\frac{1}{3}\pi r^2 h + \frac{2\pi}{3} r^3\right)$ cm^3

$= \frac{\pi r^2}{3}$ (h + 2r) cm^3

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times (6 + 2 \times 4.2)$ cm^3

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 14.4$ cm^2 = 266.11 cm^3

34. Let the smaller side of the triangle be x cm. therefore, the second side will be (x + 4) cm, and third side is (2x - 6) cm.

Now, perimeter of triangle = x (x + 4) + (2x - 6)

= (4x - 2) cm

Also, perimeter of triangle = 50 cm.

4x = 52; x = 52 ÷ 4 = 13

Therefore, the three sides are 13 cm, 17 cm, 20 cm

$s = \frac{13+17+20}{2} = \frac{50}{2} = 25cm$

$\therefore$ Area of $\Delta = \sqrt{25(25-13)(25-17)(25-20)}$

$= \sqrt{25 \times 12 \times 8 \times 5} = \sqrt{5 \times 5 \times 4 \times 3 \times 4 \times 2 \times 5}$

$= 5 \times 4 \times \sqrt{3 \times 2 \times 5} = 20\sqrt{30} cm^2$

OR

Let a, b, c be the sides of the given triangle and s be its semi-perimeter.

Then, $s = \frac{a+b+c}{2}$ ...(i)

$\therefore$ Area of the given triangle$= \sqrt{s(s-a)(s-b)(s-c)} = \Delta$ say

As per given condition, the sides of the new triangle will be 2a, 2b, and 2c.

So, the semi-perimeter of the new triangle =

$s' = \frac{2a+2b+2c}{2} = a + b + c$ ...(ii)

From (i) and (ii), we get

s' = 2s

Area of new triangle $= \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

The required ratio = $4\Delta : \Delta$ = 4:1

Therefore the ratio of area of new triangle thus formed and the given triangle is 4 : 1.

35. Here, $f(x) = x^4 + px^3 + 2x^2 - 3x + q$

$g(x) = x^2 - 1$

first, we need to find the factors of $x^2 - 1$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$\Rightarrow$ (x + 1) and (x - 1)

From factor theorem, if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let us take, x + 1

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

Substitute the value of x in f(x)

$f(-1) = (-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q$

= 1 - p + 2 + 3 + q

= -p + q + 6

Since f(-1)=0, therefore,we have,

-p+q+6=0...........(1)

Let us take, x - 1

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Substitute the value of x in f(x)

$f(1) = (1)^4 + p(1)^3 + 2(1)^2 - 3(1) + q$

= 1 + p + 2 - 3 + q

= p + q

Since f(1)=0, therefore, we have,

f(1)=0, therefore, we have,

p+q=0............(2)

Solve equations 1 and 2

- p + q = - 6

p + q = 0

2q = - 6

q = - 3

substitute q value in equation 2

p + q = 0

p - 3 = 0

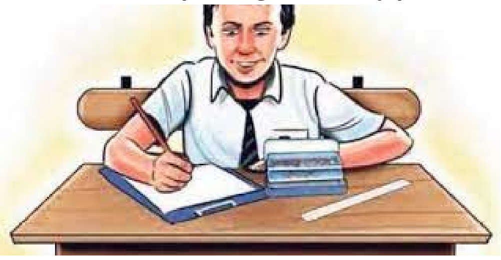
p = 3

value of p = 3 and q = – 3

Section E

36. **Read the text carefully and answer the questions:**

Ajay is writing a test which consists of ‘True’ or ‘False’ questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. Ajay knew answers to some of the questions. Rest of the questions he attempted by guessing.



(i) Let the no of questions whose answer is known to Ajay be x and number questions attempted by guessing be y.

x + y = 110

x + 14y = 80 $\Rightarrow$ 4x + y = 320x + y = 110 ...(1)

4x + y = 320 ...(2)

Solving (1) and (2)

x + y - 4x - y = 110 - 320 = -210

$\Rightarrow$ -3x = -210

$\Rightarrow$ x = 70

(ii) x + y = 110

x + 14y = 80 ⇒ 4x + y = 320

x + y = 110 ...(1)

4x + y = 320 ...(2)

Solving (1) and (2)

x + y - 4x - y = 110 - 320 = - 210

⇒ - 3x = - 210

⇒ x = 70

Put x = 70 in (1)

70 + y = 110

⇒ y = 40

40 question he answered by guessing.

(iii) $70 - 40 \times \frac{1}{4} = 70 - 10 = 60$ marks

He scored 60 marks.

OR

$x - \frac{1}{4}(110 - x) = 95$

⇒ 4x - 110 + x = 380

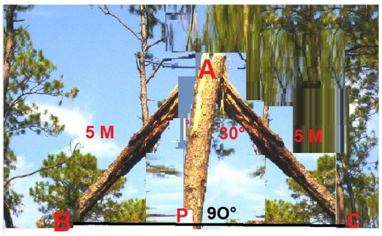
⇒ 5x = 380 + 110 = 490

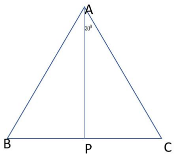
⇒ $x = \frac{490}{5} = 98$

So he answered 98 correct answers 12 by guessing.

37. **Read the text carefully and answer the questions:**

In a forest, a big tree got broken due to heavy rain and wind. Due to this rain the big branches AB and AC with lengths 5m fell down on the ground. Branch AC makes an angle of 30° with the main tree AP. The distance of Point B from P is 4 m. You can observe that ΔABP is congruent to ΔACP.





(i) In △ACP and △ABP

AB = AC (Given)

AP = AP (common)

$\angle APB = \angle APC = 90^o$

By RHS criteria △ACP ≅ △ABP

(ii) In △ACP

$\angle APC + \angle PAC + \angle ACP = 180^o$

⇒ $90^o + 30^o + \angle ACP = 180^o$ (angle sum property of △)

⇒ $\angle ACP = 180^o - 120^o = 60^o$

$\angle ACP = 60^o$

(iii) △ACP ≅ △ABP

Corresponding part of congruent triangle

∠BAP = ∠CAP

$\angle BAP = 30^o$ (given $\angle CAP = 30^o$)

OR

△ACP

$AC^2 = AP^2 + PC^2$

⇒ $25 = AP^2 + 16$

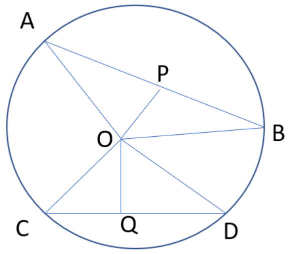
⇒ $AP^2 = 25 - 16 = 9$

⇒ AP = 3

Total height of the tree = AP + 5 = 3 + 5 = 8 m

38. **Read the text carefully and answer the questions:**

Rohan draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.



(i) In ΔAOP and ΔBOP

$\angle APO = \angle BPO$ (Given)

OP = OP (Common)

AO = OB (radius of circle)

$\Delta AOP \cong \Delta BOP$

AP = BP (CPCT)

(ii) In right ΔCOQ

$CO^2 = OQ^2 + CQ^2$

$\Rightarrow 10^2 = 8^2 + CQ^2$

$\Rightarrow CQ^2 = 100 - 64 = 36$

$\Rightarrow CQ = 6$

CD = 2CQ

$\Rightarrow$ CD = 12 cm

(iii) In right ΔAOB

$AO^2 = OP^2 + AP^2$

$\Rightarrow 10^2 = 6^2 + AP^2$

$\Rightarrow AP^2 = 100 - 36 = 64$

$\Rightarrow AP = 8$

AB = 2AP

$\Rightarrow$ AB = 16 cm

OR

There is one and only one circle passing through three given non-collinear points.